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Effect of collisional plasma on the transient response of electromagnetic pulses

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The transient response of short duration electromagnetic pulses reflected from collisional plasma has been studied. With the help of expressions obtained for the real and imaginary parts of reflected electromagnetic field vectors, the polarization characteristics have been studied. It is shown that the change in the polarization characteristics is of significant diagnostic value.

1. Introduction

It is well known that the transmission and reflection characteristics of electromagnetic waves are governed by the intervening external fields and plasma parameters (Schmitt 1964, 1965, Knop 1964) The measurements of the emergent and reflected electromagnetic wave characteristics are often used to estimate the relevant external field and ambient parameters of extended and bounded plasma. The response of rarefied, isotropic, cold and magnetic field-free plasma to electromagnetic pulse propagation is well known. However, in the case of anisotropic and hot plasmas, the response is comparatively less predictable. The nature of the electromagnetic pulse response during transmission and reflection processes is recorded and interpreted in each case to study the detailed structure of the intervening plasma. The response of the plasma becomes pronounced when the incident electromagnetic pulses are short compared to the periodic time of various microscopic features of the plasma. Under these conditions the electromagnetic pulses incident on a plasma system give rise to a characteristic transient ringing effect which is typical of the ratios of the incident wave and ambient plasma parameters. The oscillations of the plasma system initially starts at a low frequency and finally stablises at the plasma frequency. Due to finite time lag between excitation and stabilization of plasma oscillation and wave-wave interaction the emergent pulse is distorted which gives rise to varying transient response of the plasma system.

The cumulative transient response of the intervening extended collisional plasma system on the short duration electromagnetic pulse is obtained by synthesizing the signal using time convolution technique. The expressions for electromagnetic field components reflected from collisional plasma have been obtained in terms of real frequency component. For quantitative study of the formulation we have chosen ionospheric plasma parameters and have computed the transient response of short duration electromagnetic pulses. The response of a fixed radio frequency pulse from two plasma densities and same collision frequency have been computed and are shown. It is argued that the changes in polar characteristics of reflected transient response of short duration radio frequency pulses arise due to changing wave frequency and plasma parameters.

2. Theoretical Considerations

Case & Haskell (1966) studied the transient response of vertically reflected electromagnetic pulses from magnetoplasma and obtained an expression for the relative dielectric constant

$$K^{\pm} = 1 + \frac{\omega_p^2}{S[S \mp i\omega_c]}\,. \qquad \ldots \quad (1)$$

The plus and minus signs refer to two modes of wave propagation in the magnetoplasma. The ω_p and ω_c are electron plasma and electron cyclotron frequencies in radian per second. The transient response analysis of a complex frequency signal $S = i\omega$ reflected from collisionless plasma was carried out by Case & Haskell (1966). The non-vertical incidence and reflection of electromagnetic pulses can be easily accounted for by modifying eq. (1). The similar analysis in terms of dimensionless wave and plasma parameters and accounting for the effect of finite interparticle collisions is presented in this paper. The corresponding relative dielectric constant is written as

$$K^{\pm} = 1 + \frac{\delta^2}{\eta[\eta \mp i\delta_c + Z]}, \qquad \ldots \quad (2)$$

where the complex frequency notation used in eq. (1) has been expressed as $S = i(\omega_1 + i\omega_2)$ and the resulting dimensionless parameters are defined as

$$\delta = \frac{\omega_p}{\omega_1},\ \delta_c = \frac{\omega_c}{\omega_1},\ Z = \frac{\nu}{\omega_1},$$

and dimensionless complex variable

$$\eta = i\left(1 + i\frac{\omega_2}{\omega_1}\right).$$

Taking these notations into account the reflection coefficient in terms of dimensionless parameters is written as

$$R^{\pm}=\frac{1-\sqrt{K^{\pm}}}{1+\sqrt{K^{\pm}}},$$

$$=\frac{\sqrt{\eta(\eta\mp i\delta_c+Z)}-\sqrt{\eta(\eta\mp i\delta_c+Z)+\delta^2}}{\sqrt{\eta(\eta\mp i\delta_c+Z)}+\sqrt{\eta(\eta\mp i\delta_c+Z)+\delta^2}}. \tag{3}$$

With the help of inverse Laplace transform the plasma response, which is function of dimensionless complex variable, is changed into real variable. The transient response of short duration electromagnetic pulses reflected from anisotropic, collisional and warm plasma medium is easily obtained by taking the inverse Laplace transform of the reflection coefficient. The nature of the resulting reflected signal is mainly governed by the real part of the incident frequency ω_1 and time. In order to take inverse Laplace transform the reflection coefficient in eq. (3) is written as

$$R^{\pm}=\frac{\sqrt{\left(\eta+\frac{\mp i\delta_c+Z}{2}\right)^2+\left(\frac{\delta_c\pm iZ}{2}\right)^2}-\sqrt{\left(\eta+\frac{\mp i\delta_c+Z}{2}\right)^2+\left(\frac{\delta_c\pm iZ}{2}\right)^2+\delta^2}}{\sqrt{\left(\eta+\frac{\mp i\delta_c+Z}{2}\right)^2+\left(\frac{\delta_c\pm iZ}{2}\right)^2}+\sqrt{\left(\eta+\frac{\mp i\delta_c+Z}{2}\right)^2+\left(\frac{\delta_c\pm iZ}{2}\right)^2+\delta^2}} \tag{4}$$

Now using Laplace transform identities as used by Case & Haskell (1966) we obtain the inverse Laplace transform of eq. (4) as

$$L^{-1}[R^{\pm}]=\delta\left[-\frac{2J_2(\tau)}{\tau}+\frac{\delta_c\pm iZ}{\delta}\int_0^{\tau}J_2\frac{(\sqrt{\tau^2-x^2}}{\sqrt{\tau^2-x^2}}\,J_1\left\{\left(\frac{\delta_c\pm iZ}{2}\right)x\right\}dx\right]$$

$$\times\exp\left(\frac{i\delta_c}{2}\cdot\alpha\right)\exp\left(-\frac{Z\alpha}{2}\right),$$

$$\tau=\delta\alpha,\quad x=\delta\theta,\quad a\pm ib=\frac{\delta_c\pm iZ}{2\delta} \tag{5}$$

The reflected signal response corresponding to a pulse of unit amplitude and duration T which is short as compared to the periods of the plasma and cyclotron frequencies is obtained by using the time convolution

$$E_R^{\pm}=TL^{-1}[R_R^{\pm}]=T\delta f(\tau)\exp\left(\mp\frac{i\delta_c}{2}\alpha\right)\exp\left(-\frac{Z\alpha}{2}\right), \tag{6}$$

where

$$f(\tau)=\left[-\frac{2J_2(\tau)}{\tau}+2(a\pm ib)\int_0^{\tau}\frac{J_2(\sqrt{\tau^2-x^2})}{\sqrt{\tau^2-x^2}}J_1\{(\alpha\pm ib)x\}dx\right]. \tag{7}$$

The eqs (6) and (7) obtained for collisional plasma when compared with corresponding equations of Case & Haskell (1966) show additional features. The reflected response is found to damp with increasing collision frequency. The magnitude of the two components of reflected signal is found to change in the presence of collisions. The argument of the Bessel function J_1 in eq. (7) becomes complex. Separating the parameter in the argument into real and imaginary parts and using the transformation equation the field components are written as

$$E_{1R} = 2\delta T \left[f(\tau) \exp\left(\mp i\frac{d_c}{2}\,\alpha\right)\right]_{R_e} \exp\left(-\frac{Z\alpha}{2}\right), \qquad \ldots (8)$$

$$E_{2R} = 2\delta T \left[f(\tau) \exp\left(\mp i\frac{\delta_c}{2}\,\alpha\right)\right]_{I_m} \exp\left(-\frac{z\alpha}{2}\right).$$

The variation of E_{1R} and E_{2R} with time governs the change in the polarization characteristics of reflected electromagnetic pulses. The applicability of these equations for collisionless laboratory plasma was discussed by Case & Haskell (1966). In the limiting case of $\alpha << 1$ the reflected response of electromagnetic wave on time scales long and short compared to a period of plasma oscillation would change. Th cumulative change in the normalised polarization characteristic of the reflected signal have been computed using eqs (8) and (9). Figure (1) shows the plot of $E_{1R}/2\delta T$ versus $E_{2R}/2\delta T$ for radio frequency pulse incident on the

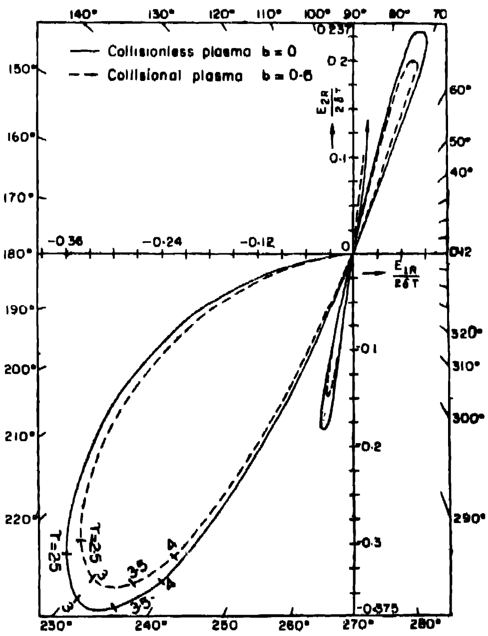

Fig. 1. Temporal evolute of reflected electric field components. $a = 0{,}06$, $\nu = 3\times10^6$ rad/sec and $\omega_1 = 7.7\times10^7$ rad/sec

ionospheric plasma. The change in the polarization features arising due to changes in the plasma frequency is depicted by comparison of figures 1 and 2. The solid curves in both these figures depict the role of collisionless plasma whereas the

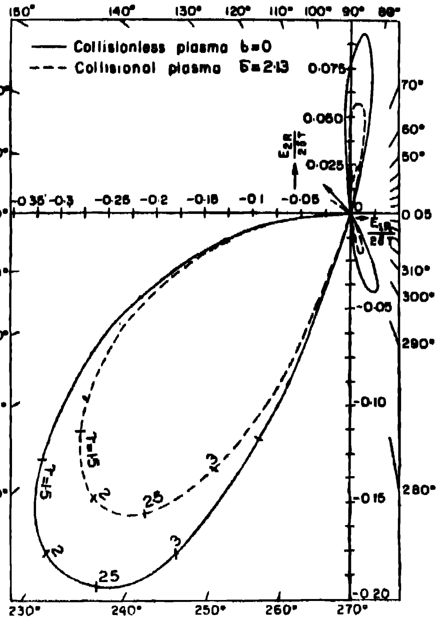

Fig. 2. Temporal evoluto of reflected electric field components. a = 0.2125, 3×10^{6} rad/sec and $\omega_1 = 1.66\times10^{7}$ rad/sec.

dotted curves depict the role of collisions. The maximum change of polarization angle of 4° corresponds to a collision frequency of 3×10^{6} rad/sec. By using polarization sensitive antenna system and appropriate receiver a change of polarization of a fraction of degree can be measured. Therefore, this technique may provide an estimate of fast and large variations in electron density and collision frequency of the reflecting region. The collision frequency for the curves has been kept constant and the propagation time has been changed. The polar diagram thus obtained shows the time rate of change in polarization. With the increase of collision frequency the dotted curve shrinks and the corresponding angle also changes. The change in polarization angle with time is shown by the polar diagram. The values of τ are shown for several points of the curve (figures 1, 2). Further, the expressions for dielectric constant change from extended gaseous to bounded gaseous and solid state plasma. The parallel analysis for a chosen plasma system gives similar information. The reflection of microwaves, millimeter and sub-millimeter waves from solid state plasma would reveal corresponding changes in the polarization characteristics. The precise measurement of polarization characteristics of short duration pulses would also serve as a diagnostic technique for solid state plasma parameters such as number density and collision frequency.

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